Coexistence of Pareto Efficiency and False-name-proofness in Social Choice

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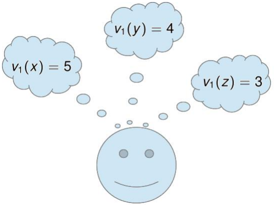
Social Choice Function

Multiagent setting:

- Set of identities I
- Identities involved $N \subseteq I$
- Set of solutions X
- Set of utility functions $(v_i)_{i \in N}$
- Set of utility domains $(V_i)_{i \in N}$

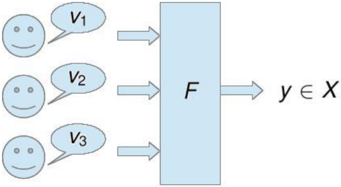


Notations:

- $v = (v_1, \ldots, v_n)$
- $V_N = \prod_{i \in N} V_i$

Social Choice Function

- $F : \bigcup_{N \subseteq I} V_N \to X$



Social Welfare Function

- $f : X \times \bigcup_{N \subseteq I} V_N \to \mathbb{R}$ s.t. $F(v) \in \arg\max_{y \in X} f(y, v)$

Social welfare maximizer (Pareto efficient)

$$f_\Sigma(x, v) = \sum_{i \in N} v_i(x)$$

EXAMPLE

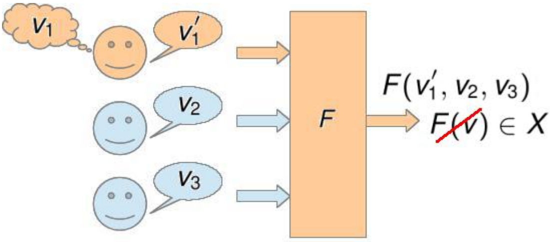
Example with $N = \{1, 2, 3\}$ and $X = \{x, y, z\}$:

	v_1	v_2	v_3
x	5	3	2
y	4	4	3
z	3	2	5

- $f_\Sigma(x, v) = 5 + 3 + 2 = 10$
- $f_\Sigma(y, v) = 4 + 4 + 3 = 11$
- $f_\Sigma(z, v) = 10$

$$F(v) = y$$

MANIPULATIONS



	(v_1)	v_1'	v_2	v_3
x	(5)	6	3	2
y	(4)	3	4	3
z	(3)	3	2	5

$$F(v_1', v_2, v_3) = x$$

Notations:

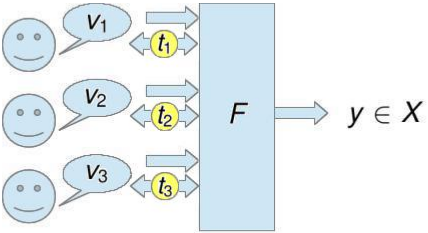
- $v_{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$
- $V_{-i} = \prod_{j \in N \setminus \{i\}} V_j$

Set of transfert functions $t = (t_i)_{i \in I}$

- $t_i : X \times \bigcup_{N \subseteq I \setminus \{i\}} V_N \to \mathbb{R}$
- $u_i(v_{-i}, x) = v_i(x) + t_i(v_{-i}, x)$

Mechanism

- $\{F, t\}$



Definition

A mechanism (F, t) is said to be incentive compatible if $\forall N \subseteq I, \forall v \in V_N, \forall i \in N, \forall v'_i \in V_i$ *we have:*

$$v_i(F(v) + t_i(F(v), v_{-i}) \geq v_i(F(v'_i, v_{-i})) + t_i(F(v'_i, v_{-i}), v_{-i})$$

Unrestricted domain:

- $\forall i \in I$, $V_i = \mathbb{R}^X$

Definition

A mechanism (F, t) is called Groves mechanism if for any $N = \{1, \ldots, n\} \subseteq I$ there is some functions $h_1, \ldots, h_n$ where $\forall i \in N$, $h_i : V_{N \setminus \{i\}} \to \mathbb{R}$ is such that:

$$t_i(F(v), v_{-i}) = \sum_{j \in N \setminus \{i\}} v_j(F(v)) + h_i(v_{-i})$$

Definition

We say that a mechanism (F, t) *is Individualy Rational (IR) whenever* $\forall N \subseteq I, \forall v \in V_N, \forall i \in N$ *we have:*

$$v_i(F(v)) + t_i(F(v), v_{-i}) \geq v_i(F(v_{-i}))$$

Definition

A mechansim (F, t) *is called VCG mechanism if it is a Groves mechanism s.t.* $\forall N \subseteq I, \forall v \subseteq V_N, \forall i \in N$ *the function* h_i *for any* $v_{-i} \in V_{-i}$ *is defined as*

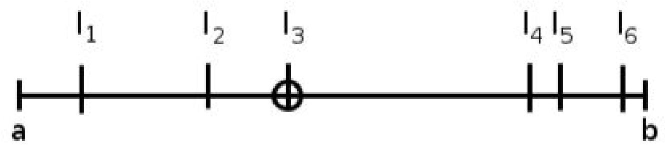
$$h_i(v_{-i}) = \max_{x' \in X} \sum_{j \in N \setminus \{i\}} v_j(x')$$

- VCG mechanism are individualy rational

Illustrative example of false-name-manipulation

Facility location problem on a staight line

- segment $[a, b]$
- $\forall i \in N, l_i \in [a, b]$ s.t. $\forall i \in N \backslash \{n\}, l_i \leq l_{i+1}$
- location $d \in [a, b]$
- $\forall j \in N, v_j(d) = |d - l_j|$
- $F(v) = l_{\lfloor \frac{n}{2} \rfloor}$



Illustrative example of false-name manipulation

Facility location problem on a staight line

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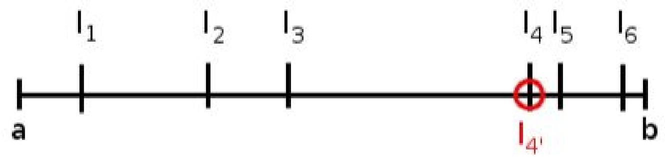


Definition

A mechanism (F, t) *is said to be false-name-proof if*
$\forall N \subseteq I, \forall v \in V_N, B \subseteq N, \forall i \in B$ *and* $\forall v_i' \in V_i$ *we have:*

$$v_i'(x) + t_i(x, v_{-B}) \geq v_i'(F(v)) + \sum_{j \in B} t_j(F(v), v_{-j})$$

where $x = F(v_i', v_{-B})$.

Question: can we design a mechanism with at the same time the properties of false-name-proofness and individual rationality for a given domain?

Definition

A preference domain V_I is said to be symmetric whenever there exists $D \subseteq R^X$ such that for any $i \in I$ we have $V_i = D$.

Definition

A preference domain V_I is said to be competitive whenever there exists $x, y \in X$ such that $\forall i \in N$ we can find $v_i^x, v_i^y \in V_i$ such that $\forall z \in X \backslash \{x\}, v_i^x(x) > v_i^x(z)$ and $\forall z \in X \backslash \{y\}, v_i^y(y) > v_i^y(z)$.

Proposition

Whenever the preference domain is symmetric and competitive, false-name-proofness and individual rationality are incompatible for the social welfare maximizer.

Proposition

A Groves mechanism defined for any $N \subseteq I$, any $v \in V_N$ and any $i \in N$ by the following function

$$h_i(v_{-i}) = -\sum_{j \in N \setminus \{i\}} \max_{x' \in X} \{v_j(x')\}$$

is false-name-proof for any domain.

$$t_i(F(v), v_{-i}) = \sum_{j \in N \setminus \{i\}} [v_j(F(v)) - \max_{x' \in X} \{v_j(x')\}]$$

Definition

The distance of a mechanism (F, t) to individual rationality is defined for a given $N \subseteq I$ and a given $v \in V_N$ by the following fonction:

$$d(v) = \max_{i \in N} \{d_i(v)\}$$

where for any $i \in N$

$$d_i(v) = v_i(F(v_{-i})) - v_i(F(v)) - t_i(F(v), v_{-i})$$

$$\Delta(v) = \max_{j \in N} \left\{ \max_{x' \in X} \{v_j(x')\} - \min_{x'' \in X} \{v_j(x'')\} \right\}$$

Proposition

For the first mechanism, the distance to individual rationality for a given $N \subseteq I$ and a given $v \in V_N$ is never greater than $(n-1)\Delta(v_{-i})$. Furthermore in the unrestricted domain case we can find some preference profile where this bound is attained.

Proposition

A Groves mechanism defined for any $N \subseteq I$, for any $v \in V_N$ and any $i \in N$ by the following function

$$h_i(v_{-i}) = -\max_{x' \in X}\{\sum_{j \in N \setminus \{i\}} v_j(x')\} - \Delta(v_{-i})$$

is false-name-proof for any domain.

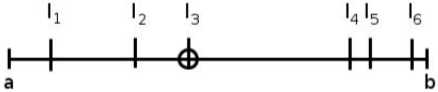
Proposition

For the second mechanism, the distance to individual rationality for a given $N \subseteq I$ and a given $v \in V_N$ is never lower than $\Delta(v_{-i})$.

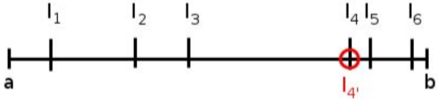
But we can find some preference profiles where the individual rationality is not violated by the first mechanism but is violated by the second mechanism.

Facilty location and first mechanism

$$t_i(d, v) = -\sum_{j \in N \setminus \{i\}} |l_j - d|$$



$$v_4(l_3) + t_4(l_3, v_{-4}) = -\sum_{j \in N} |l_j - l_3|$$

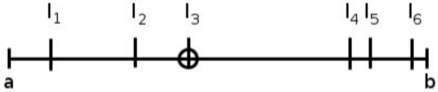


$$v_4(l_4) + t_4(l_4, v_{-4}) + t_{4'}(l_4, v_{-4'}) = -2\sum_{j \in N} |l_j - l_4|$$

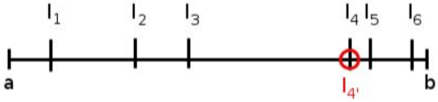
Lemma

The values of the transfer function of the VCG mechanism for the facility location problem on a line are 0 when the size of N is odd. Furthermore if the size of N is pair then the value of the transfer function for an agent in $\{1, \ldots, \frac{n}{2}\}$ *is* $l_{\frac{n}{2}} - l_{\frac{n}{2}+1}$, *and 0 for the other agents.*

$$t_i(l_{\lfloor \frac{n}{2} \rfloor}, v) = \begin{cases} \max_{j \in N \setminus \{i\}} \{\max(|l_j - a|, |b - l_j|)\} + |l_{\frac{n}{2}+1} - l_{\frac{n}{2}}| & \text{if } n \text{ pair } \& \ i \leq \frac{n}{2} \\ \max_{j \in N \setminus \{i\}} \{\max(|l_j - a|, |b - l_j|)\} & \text{otherwise} \end{cases}$$



$$v_4(l_3) + t_4(l_3, v_{-4}) = -|l_4 - l_3| - |l_6 - a|$$



$$v_4(l_4) + t_4(l_4, v_{-4}) + t_{4'}(l_4, v_{-4'}) = -2|l_6 - a|$$

Conclusion and Perspectives

Conclusion

- Incompatibility of false-name-proofness, incentive compatibility and Pareto efficiency for a wide class of social choice function
- 2 false-name-proof and efficient mechanisms
- comparisons in terms of distance to individual rationality

Perspectives

- find a better false-name-proof mechanism in terms of distance to individual rationality
- relaxing another requirement of our impossibility result